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A dynamic scattering approach for a gated interacting wire

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Abstract. A scattering approach for correlated one-dimensional systems is developed. The perfect contact to charge reservoirs is encoded in time-dependent boundary conditions. The conductance matrix for an arbitrary gated wire, respecting charge conservation, is expressed through a dynamic scattering matrix. Two applications are developed. First, it is shown that the dc conductance is equal to e^2/h for any model with conserved total left- and right-moving charges. Second, the ac conductance matrix is explicitly computated for the Tomonaga-Luttinger model (TLL).

PACS. 71.10.Pm Fermions in reduced dimensions (anyons, composite fermions, Luttinger liquid, etc.) – 72.10.-d Theory of electronic transport; scattering mechanisms – 73.23.-b Mesoscopic systems

Pioneered by Landauer[1], the scattering approach for quantum transport has proven powerful in mesoscopic physics. Nevertheless, it is restricted to non-interacting systems, and to the stationary regime. There were formal extensions to finite frequency transport based on a self-consistent approach [2], or non-equilibrium techniques for interacting dots [3], but these formalisms are difficult to exploit. Proposed here is a different scattering approach for linear ac transport through a strongly correlated, one-dimensional wire in the low-energy regime. Charge reservoirs perfectly connected to the wire are accounted for by appropriate boundary conditions. Coupling to a gate is incorporated, ensuring charge conservation. The corresponding AC 3×3 conductance matrix is expressed through a novel dynamic "scattering" matrix $\mathrm{S}(\omega)$. Further progress is then made in two cases. First, for any model where the total charge for right- and left-moving electrons is conserved, the transmission is shown to be unity in the zero-frequency limit. This generalizes the DC conductance result $g = e^2/h$ shown for a Tomonaga-Luttinger liquid (TLL) [4–6] or for arbitrary finite-range interactions [7]. A similar result was shown in reference [8] through different hypothesis and arguments restricted to the stationary regime, without describing the reservoir-wire interface. Second, $\mathrm{S}(\omega)$ is computed for the TLL model, giving an AC conductance that depends on interactions in contrast to the stationary regime.

Without connecting one-dimensional leads to an interacting wire, this work extends the concept introduced in references [4,9,10] where reservoirs are accounted for by the electrons they inject. The leads have served to define the incident and transmitted electrons, different from the proper modes of the wire. For a TLL model, the same conductance results were found by the author by computing the current in response to an appropriate external electric field [9,10]. More recently, they were confirmed by Blanter *et al.* [11] through a self-consistent treatment of interactions, justified in the absence of backscattering. Other works based on the Kubo formula in a TLL with leads found different results due to a different electric field profile to which current is very sensitive [12].

An underlying hypothesis of Landauer's approach for noninteracting systems [1] is the ideal nature of the contacts, ensuring that emerging electrons are absorbed without reflection by the reservoirs [1]. Such a concept cannot be extended to interacting systems [4,13,14]. Rather, interactions give rise to collective excitations, or Laughlin quasiparticles in edge states, that are different from the electrons in the reservoirs. An emerging "quasiparticle" undergoes a quasi-Andreev type reflection [4,9,10,15] at a perfect contact with a reservoir, and this is a key point.

This paper is mainly concerned with systems connected locally to reservoirs, such as quantum wires, or nanotubes; edge states couple differently to reservoirs [14].

Consider an arbitrary one-dimensional finite wire delimited by $[-a, a]$, whose length is

$$L = 2a.$$

The long wavelength part of the electronic density can be decomposed into right and left-moving electron densities [16] ρ_+ and ρ_- including implicitly the zero modes, $\rho = \rho_+ + \rho_-$ where spin is ignored for simplicity. For $r = \pm$, the boson field Φ_r defined by $\rho_r = -\partial_x \Phi_r / 2\pi$ is the canonical conjugate to $r\rho_r$ (Kac-Moody algebra).

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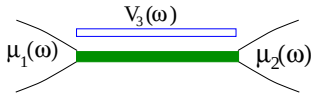

Fig. 1. A gated wire perfectly connected to charge reservoirs with time-dependent electrochemical potential $\mu_{1,2}(t) = \mathrm{e}^{\mathrm{i}\omega t}\mu_{1,2}(\omega)$. The boundary conditions apply in the presence of arbitrary backscattering inside the wire.

The kinetic Hamiltonian is $H_{\rm kin} = \int_{-a}^{a} hv_{\rm F}(\rho_+^2 + \rho_-^2)/2$. Any interaction Hamiltonian $H_{\rm int}$ either between electrons or with impurities can be expressed as a functional of Φ_+, Φ_-, thus the total Hamiltonian,

$$H = H_{\rm kin} + H_{\rm int} + eV_{\rm gate}Q = H(\rho_+, \rho_-), \qquad (1)$$

is a functional of $\rho_\pm$. Coupling to a gate is incorporated, and $Q = \int_{-a}^{a} \rho(x)$.

The current field $j(x)$ can be expressed independently of the dynamics, in or out-of-equilibrium. For this, the Hamiltonian $H^{(A)}$ in the presence of a vector potential A, is used:

$$j(x) = -\left.\frac{\delta H^{(A)}}{\delta A(x)}\right|_{A=0}. \qquad (2)$$

A can be absorbed by a gauge transformation of the right and left-going fermion fields, $\Psi_r \sim \mathrm{e}^{\mathrm{i}r\Phi_r}$ for $r = \pm$ [16]. This is accomplished by the substitution

$$\Phi_r(x) \to \Phi_r(x) - \frac{re}{\hbar}\int^x A(x')\mathrm{d}x'.$$

Taking the spatial derivative, one obtains $H^{(A)}$ as

$$H^{(A)} = H\left(\rho_+ + \frac{e}{h}A, \rho_- - \frac{e}{h}A\right).$$

Differentiating with respect to A yields [17]

$$j(x) = \frac{e}{h}\left[\mu_+(x) - \mu_-(x)\right], \qquad (3)$$

where μ_r for $r = \pm$ are operators that play a central role:

$$\mu_r(x) = \frac{\delta H}{\delta \rho_r(x)} = hv_{\rm F}\rho_r + \frac{\delta H_{\rm int}}{\delta \rho_r(x)} + eV_{\rm gate}. \qquad (4)$$

Also of use will be their average,

$$\mu(x) = \frac{\partial H}{\delta \rho(x)} = \frac{1}{2}\left[\mu_+(x) + \mu_-(x)\right]. \qquad (5)$$

One can think of the expectation value of $\mu_+(x)$ as the energy required to add a right-going electron at x. It includes the local Fermi energy $hv_{\rm F}\rho_+(x)$, the interaction energy, and the gate potential that shifts the bottom band. In some sense, it is a local electro-chemical potential for right-going electrons. Consider now a typical transport measurement, where one connects perfectly the wire at $\pm a$ to charge reservoirs (see Fig. 1). Spatial and temporal structure on the scale of $\lambda_{\rm F}$ (Friedel oscillations) are ignored. The left (right) reservoir injects bare right (left)-going electrons with a well defined electro-chemical potential μ_1 (μ_2). At the contacts where both incident fluxes impinge, the energy is conserved in the absence of any dissipation processes. Thus the field $\mu_+(-a,t)$ $[\mu_-(a,t)]$ is required to be pinned to μ_1 $[\mu_2]$ at any time. Extending this condition to alternative regimes, one has:

$$\begin{aligned}\mu_+(-a,t) &= \mu_1(t)\\ \mu_-(a,t) &= \mu_2(t).\end{aligned} \qquad (6)$$

Without interactions near the contacts, $\mu_\pm(\mp a,t) = hv_{\rm F}\rho_\pm(\mp a,t) + eV_{\rm gate}(t)$; equation (6) imposes the density for incident electrons, and generalizes the Landauer concept to AC transport with arbitrary backscattering. But in the presence of interactions, $\mu_\pm$ depend on both ρ_+ and ρ_-, thus the density for incident electrons is not imposed (in contradiction with Ref. [19]). Rather, electrons are partially reflected, giving rise to a contact resistance. Indeed, equation (6) leads to a discontinuous local electrostatic potential $V_{\rm loc}$. $eV_{\rm loc}$ follows the electro-chemical potential on the reservoir side, while

$$eV_{\rm loc}(x) = \frac{\delta[H - H_{\rm kin}]}{\delta\rho(x)} = \mu(x) - \frac{hv_{\rm F}}{2}\rho(x) \qquad (7)$$

on the interacting side [7,10]. On the right hand side, $\mu(x)$ is given by equation (5), and is a kind of local electro-chemical potential for both carriers [7]. This clarifies a confusing point in the self-consistent treatment of interactions in higher-dimensional systems, where the continuity of $V_{\rm loc}$ is expected, even though not implemented in the results. It would be interesting to define and then verify analogous conditions to equation (6). We have to stress however that no use of $V_{\rm loc}$ neither self consistent arguments is made here, even to ensure charge conservation.

In the sequel, I switch to the Fourier transform of expectation values in a time-dependent ground state, letting ω the external frequency. Time variation has to be slow enough so that the reservoirs are driven adiabatically through a sequence of equilibrium states with well defined time-dependent electro-chemical potential. Thus electrons must have enough time to adjust their average energy to a new value of the chemical potential during one cycle, thus $\omega\tau_{\rm in} \ll 1$, where $\tau_{\rm in}$ is the time equilibration in the reservoirs. In addition, such condition ensures that the transition region between the reservoirs and the wire, of length L', can be considered as local; this is because L' does not exceed $v_{\rm F}\tau_{\rm in}$, thus $\omega \ll \tau_{\rm in}^{-1} < v_{\rm F}/L'$. Note that for coherent transport through the wire, we have $L \ll v_{\rm F}\tau_{\rm in}$, thus the limit $\tau_{\rm in}^{-1}$ is lower than $v_{\rm F}/L$.

The AC conductance matrix $\mathbf{G}_3(\omega)$ can indeed be expressed formally for arbitrary higher frequencies, $\omega \ll E_{\rm F}$, which is the unique limitation arising from the long wavelength models; the low frequency regime will be specified afterwards. $\mathbf{G}_3(\omega)$ is a 3×3 matrix with $g_{\alpha\beta} = \delta I_\alpha/\delta V_\beta$ where $V_{1,2} = \mu_{1,2}/e$, $V_3 = V_{\rm gate}$, $I_{1,2} = \mp j(\mp a,\omega)$ and I_3 the gate current. A complete description with all the surrounding three-dimensional environment would be too complex. Instead, assume that all the electric field lines emerging from the wire end up on the gate, thus the latter carries an opposite charge to that on the wire $Q(\omega)$. This ensures Kirchoff's law, $\sum_\alpha I_\alpha = 0$ because, using

the continuity equation,

$$I_3(\omega) = -\mathrm{i}e\omega Q(\omega) = \int_{-a}^{a} \mathrm{i}\omega e\rho(x,\omega) \qquad (8)$$
$$= -\int_{-a}^{a} \partial_x j(x,\omega) = -I_1(\omega) - I_2(\omega).$$

On the other hand, $V_{\mathrm{gate}}(\omega)$ appears as a reference potential in equations (4,6). Thus it can be shown that the two constraints on the conductance matrix [2]

$$\sum_\alpha g_{\alpha\beta} = 0 = \sum_\beta g_{\alpha\beta}, \qquad (9)$$

are ensured. Next focus on the first 2×2 block of $\mathbf{G_3}(\omega)$ denoted $\mathbf{G_2}(\omega)$. Using equations (6, 3),

$$I_1 = -j(-a,\omega) = \frac{e}{h}\left[\mu_-(-a,\omega) - \mu_1(\omega)\right],$$
$$I_2 = j(+a,\omega) = \frac{e}{h}\left[\mu_+(-a,\omega) - \mu_2(\omega)\right]. \qquad (10)$$

In order to express I_α to linear order in $\mu_{1,2} = \mu_\pm(\mp a,\omega)$, it is sufficient to retain the linear dependence of $\mu_\pm(\pm a,\omega)$, determined by some matrix $\mathbf{S}(\omega)$,

$$\begin{pmatrix}\mu_-(-a,\omega)\\ \mu_+(a,\omega)\end{pmatrix} = \mathbf{S}(\omega)\begin{pmatrix}\mu_+(-a,\omega)\\ \mu_-(a,\omega)\end{pmatrix}. \qquad (11)$$

Combined with equation (10), this gives an interesting relation,

$$\mathbf{G_2}(\omega) = \frac{e^2}{h}\left[\mathbf{S}(\omega) - \mathbf{I}\right]. \qquad (12)$$

If the elements of $\mathbf{S}(\omega)$ are denoted as follows

$$\mathbf{S}(\omega) = \begin{pmatrix}R(\omega) & T'(\omega)\\ T(\omega) & R'(\omega)\end{pmatrix}, \qquad (13)$$

then $T(\omega)$ $[R(\omega)]$ can be viewed as the total dynamic transmission [reflection] coefficient for the incident flux from the left to the right reservoir (into the left reservoir). T' and R' play the same role for the right reservoir. Nevertheless, important differences from the usual scattering approach should be stressed. The elements of $\mathbf{S}(\omega)$ determine directly the current or density, but are nonetheless complex numbers. $\mathbf{S}(\omega)$ is not unitary, and in general not symmetric unless there is a perfect reflection symmetry. In addition, $T(\omega) + R(\omega) \neq 1$, and current conservation is ensured by the gate (Eq. (8)). The total conductance matrix can now be expressed, using equation (12), and letting the ω dependence be implicit:

$$\mathbf{G}_3 = \frac{e^2}{h}\begin{pmatrix}R-1 & T' & 1-R-T'\\ T & R'-1 & 1-R'-T\\ 1-R-T & 1-R'-T' & R+R'+T+T'-2\end{pmatrix} \qquad (14)$$

At zero frequency, $S(0)$ becomes real symmetric, and $T(0)+R(0) = T'(0)+R'(0) = 1$, but $R(0)$ can be negative. The DC conductance is given by the stationary transmission,

$$g = g_{12} = -g_{11} = T(0)\frac{e^2}{h}\cdot \qquad (15)$$

As a first application of these boundary conditions, consider now a model where both total charges

$$Q_\pm = \int_{-a}^{a} \rho_\pm(x)\mathrm{d}x$$

are conserved,

$$[Q_\pm, H] = 0. \qquad (16)$$

Then it is shown here that $T(0) = 1$ [18]. In the Heisenberg representation, an operator O evolves according to

$$\mathrm{i}\hbar\frac{\mathrm{d}O}{\mathrm{d}t} = [H,O] + \mathrm{i}\hbar\frac{\partial O}{\partial t},$$

but $\partial O/\partial t = 0$ in the stationary regime. For $r = \pm$, Φ_r is the canonical conjugate to ρ_r, thus $\mathrm{d}\Phi_r/\mathrm{d}t = -r\mu_r/\hbar$ (Eq. (4)). Then

$$h\frac{\mathrm{d}\rho_r}{\mathrm{d}t} = r\partial_x\mu_r$$

is an equation for field operators that one can integrate between $-a$ and a to get, using equation (16),

$$\mu_r(a,t) - \mu_r(-a,t) = rh\frac{\mathrm{d}Q_r}{\mathrm{d}t} = 0. \qquad (17)$$

On the other hand, the field $\mu_+(-a,t)$ cannot fluctuate but is equal to μ_1 (Eq. (6)), thus

$$\mu_+(a,t) = \mu_1$$

at all times. Similarly, $\mu_-(-a,t) = \mu_-(a,t) = \mu_2$. Thus $T(0) = 1$ and $R(0) = 0$ (see Eq. (11)), and the DC conductance is equal to

$$g = \frac{e^2}{h}$$

(see Eq. (15) or simply Eq. (3)).

A second application is to investigate dynamic transport in the simplest model (verifying Eq. (16)): the TLL model. The matrix $\mathbf{S}(\omega)$ in equation (11) can be computed in an instructive way, through a "transfer" matrix $\mathbf{A}(\omega)$ such that

$$\boldsymbol{\mu}(a,\omega) = \mathbf{A}(\omega)\boldsymbol{\mu}(-a,\omega), \qquad (18)$$

where $\boldsymbol{\mu}$ stands for the vector (μ_+,μ_-). For this, it is convenient to use the right- and left-propagating current modes $j_\pm$, corresponding to up and down edge-excitations in a Hall bar [20], that can be denoted "quasiparticles". The Hamiltonian is now given by

$$H = \int_{-a}^{a}\mathrm{d}x\frac{h}{2e^2uK}\left(j_+^2 + j_-^2\right) + eV_{\mathrm{gate}}Q, \qquad (19)$$

where u and K are interaction parameters [16]. Without interactions, $j_\pm = ev_{\mathrm{F}}\rho_\pm$, $u = v_{\mathrm{F}}$ and $K = 1$. $j_\pm$ propagate freely at the sound velocity u, thus

$$\mathbf{j}(a,\omega) = \mathrm{e}^{\mathrm{i}\boldsymbol{\sigma}_z\omega t_L}\mathbf{j}(-a,\omega), \qquad (20)$$

where $t_L = L/u$ is the transit time of the wire and $\boldsymbol{\sigma}_z$ the z Pauli matrix. On the other hand, $j_\pm$ are related to $\rho_\pm$

by simple diagonalization, but their relation to $\mu_\pm$ is of more use here:

$$\mathbf{j}(x,\omega) = \frac{e}{h}\mathbf{M}\boldsymbol{\mu}(x,\omega)$$
$$\mathbf{M} = \frac{1}{1+\gamma}\begin{pmatrix} 1 & -\gamma \\ -\gamma & 1 \end{pmatrix}, \qquad (21)$$

where the coefficient γ is given by [4]

$$\gamma = \frac{1-K}{1+K}. \qquad (22)$$

Then $\mathbf{M}^{-1}$ can be obtained from $\mathbf{M}$ by $\gamma \to -\gamma$. Equations (21,20) yield the "transfer" matrix

$$\mathbf{A}(\omega) = \mathbf{M}^{-1}\mathrm{e}^{\mathrm{i}\boldsymbol{\sigma}_z\omega t_L}\mathbf{M}. \qquad (23)$$

This allows to deduce the scattering matrix $\mathbf{S}(\omega)$ in equation (11), symmetric due to the mirror symmetry,

$$T(\omega) = T'(\omega) = (1-\gamma)\frac{\mathrm{e}^{-\mathrm{i}\omega t_L} + \gamma\mathrm{e}^{\mathrm{i}\omega t_L}}{\mathrm{e}^{-\mathrm{i}\omega t_L} - \gamma^2\mathrm{e}^{\mathrm{i}\omega t_L}}, \qquad (24)$$

$$R(\omega) = R'(\omega) = \gamma\left[1 - \mathrm{e}^{\mathrm{i}\omega t_L}T(\omega)\right]. \qquad (25)$$

One can check that $|\mathrm{Det}\,\mathbf{S}(\omega)| = 1$, a constraint that can be shown to hold for any quadratic Hamiltonian with time-reversal symmetry. Note that $\mathbf{S}(\omega)$ depends solely on the intrinsic properties of the TLL model; the boundary conditions (6) allow to express the AC conductance matrix through $\mathbf{S}(\omega)$, equation (14). I now analyze in more details the capacitive effects. The gate conductance is

$$g_{33}(\omega) = 2\frac{e^2}{h}(1-\gamma)\frac{1-\mathrm{e}^{\mathrm{i}\omega t_L}}{1+\gamma\mathrm{e}^{\mathrm{i}\omega t_L}}. \qquad (26)$$

Thus the "electro-chemical" capacitance of the wire per unit length with respect to the gate [10,11]

$$C = -\lim_{\omega\to 0}\left[\frac{g_{33}(\omega)}{\mathrm{i}\omega L}\right] = 2\frac{K}{u}\frac{e^2}{h}, \qquad (27)$$

is proportional to the compressibility [16]. This result can be checked by minimizing the zero mode contribution to H (Eq. (19)) at fixed total current, thus by minimizing

$$\frac{hu}{4LK}Q^2 + eQV_{\mathrm{gate}}.$$

C results from two capacitors in series: its value without interactions $C_0 = e^2\mathrm{d}n/\mathrm{d}E = e^2h v_\mathrm{F}/2$, of purely kinetic origin, and the "electrostatic" capacitance c, obtained by evaluating equation (7) [7,11,21],

$$c = e\frac{\delta\rho}{\delta V_{\mathrm{loc}}} = \frac{e^2}{h}\left(\frac{u}{K} - v_\mathrm{F}\right)^{-1}. \qquad (28)$$

$$\frac{1}{C} = \frac{1}{c} + \frac{1}{C_0}. \qquad (29)$$

Interestingly, evaluating then differentiating equations (7, 5) with respect to ρ allows to recover equation (29). It is worth noting that

$$\mu(x) = \frac{e^2}{C}\rho(x) + eV_{\mathrm{gate}}, \qquad (30)$$

where C is given by equation (27), justifying its interpretation as a local electrochemical potential for both carriers in reference [7]. But measuring C gives the ratio K/u, leaving both u and K unknown. In the regime $\omega t_L \ll 1$, the non-dissipative part [10,11] of $g_{33}(\omega)$ (Eq. (26)) is

$$-\mathrm{Im}[g_{33}(\omega)] \simeq X - \frac{X^3}{16}\left(1 - \frac{1}{3K^2}\right), \qquad (31)$$

with $X = CL\omega$. In reference [11], the rapid variation on K of $1 - 1/3K^2$ was proposed as a way to measure K. An alternative strategy consists in measuring the leading term X on the right hand side of equation (31), then the subleading term that one divides by X^3 to infer $(1 - 1/3K^2)/16$, thus K. u can be then determined from C, equation (27).

The underlying dynamics are now interpreted. Equation (21) is equivalent to

$$\begin{pmatrix} e\mu_-(x,\omega)/h \\ j_+(x,\omega) \end{pmatrix} = \begin{pmatrix} \gamma & 1+\gamma \\ 1-\gamma & -\gamma \end{pmatrix}\begin{pmatrix} e\mu_+(x,\omega)/h \\ j_-(x,\omega) \end{pmatrix}, \qquad (32)$$

so that the matrix on the right hand side can be viewed as a local "scattering" matrix [4,9]. Let us focus for instance on $x = -a$ where $\mu_+(-a,\omega) = \mu_1(\omega)$. When no charge is incident from the left reservoir, *i.e.* $\mu_1 = 0$, then $j_+ = -\gamma j_-$; $-\gamma$ is the reflection coefficient for a "quasiparticle" incident on the contact. For repulsive interactions, $K < 1$, thus $-\gamma < 0$; a "quasi-hole" is reflected, in analogy with Andreev reflection [4,9,10,15]. If no "quasiparticle" comes from the right, *i.e.* $j_- = 0$, then one finds

$$j_+ = \frac{e}{h}(1-\gamma)\mu_1, \qquad (33)$$

and thus $1 - \gamma = 2K/(1+K)$ is the transmission coefficient for the incident flux from the reservoir. $T(\omega)$ and $R(\omega)$ result from the multiple reflections on the contacts, in analogy with a Fabry-Perot resonator [4,22]. They have resonances at the collective modes of the finite wire $\omega_n = uq$ for $q = 2n\pi/2a$, at which $T = 1$ and $R = 0$. Note that since $j = j_+ - j_-$, equation (33) obtained for $j_- = 0$ yields the current at the interface of a semi-infinite TLL and a Fermi liquid, the DC conductance becomes $g_a = (e^2/h)2K/(1+K)$ [4,9,14].

Indeed, the above scattering matrices have been encountered in references [4,9,10] (though their present interpretation was not given) where a TLL is connected perfectly to noninteracting leads at $\pm a$. There, the corresponding Hamiltonian has space dependent interaction parameters that coincide with their noninteracting values for $|x| > a$, $u = v_\mathrm{F}$ and $K = 1$. This connexion is clarified in the sequel.

If an electron impinges at $t = 0$ on $-a$, *i.e.* $\rho_+(x, t = 0) = \delta(x + a)$, the transmitted (respectively reflected) charge to a (at $-a$) at time t, *i.e.* $\rho_+(a,t)$ ($\rho_-(-a,t)$) is given by the function $M_{++}(a,-a,t)$ ($M_{-+}(-a,-a,t)$) whose Fourier transform can be shown to *coincide* exactly with $T(\omega)$ (respectively $R(\omega)$) in equation (32),

$$T(\omega) = \int \mathrm{d}t\mathrm{e}^{\mathrm{i}\omega t}M_{++}(-a,a,t) \qquad (34)$$

$$R(\omega) = \int \mathrm{d}t \mathrm{e}^{\mathrm{i}\omega t} M_{-+}(-a,-a,t).$$

In addition, these functions determine the non local dynamic conductivity at the contacts [4,9],

$$\sigma(a,-a,\omega) = \frac{e^2}{h} T(\omega)$$
$$\sigma(a,a,\omega) = \frac{e^2}{h}[1-R(\omega)]. \quad (35)$$

The reservoirs can be modeled by an *external* potential that drops only at the contacts [9,10], so that, taking into account the constant gate potential in $[-a,a]$,

$$E(x,\omega) = [V_1 - V_{\mathrm{gate}}]\,\delta(x+a) - [V_2 - V_{\mathrm{gate}}]\,\delta(x-a).$$

Then using $j(x,\omega) = \int \sigma(x,y,\omega)E(y,\omega)\mathrm{d}y$, one gets

$$j(\pm a,\omega) = \sigma(a,\mp a,\omega)\,[V_{1,2}(\omega) - V_{\mathrm{gate}}(\omega)]\,,$$

leading to the same $\mathbf{G_3}(\omega)$ (Eq. (14)) by use of equations (35, 8) [9,10].

But this is not a pure coincidence. The action being quadratic, the ground state properties are given exactly by minimizing it. Letting the ω dependence implicit, the equation of motion thus obtained imposes the continuity [4,7] of both $j = e\,[\mu_+ - \mu_-]\,/h$ and that of $\mu = (\mu_+ + \mu_-)/2$, equation (5) (expressed through Eq. (30) in the TLL). Thus both μ_+ and μ_- are continuous. On the other hand, for any $|x| > a$, $\mu_\pm(x) = hv_{\mathrm{F}}\rho_\pm(x)$ (see Eq. (4)). The left reservoir injects electrons with density $\rho_+(-a^{(-)},\omega) = \mu_1(\omega)/hv_{\mathrm{F}}$ on the noninteracting lead side, thus $\mu_+(-a^{(-)},\omega) = \mu_1(\omega)$. This fixes the continuous field μ_+ on the interacting side, $\mu_+(-a^{(+)},\omega) = \mu_1(\omega)$, which is exactly equation (6). Similar reasoning holds symmetrically for the right reservoir. Also μ_-/h has to be continuous at $-a$, thus it is equal to $v_{\mathrm{F}}\rho_-$, the reflected current. All the analysis of the pure TLL connected to leads can be extended to an arbitrary quadratic Hamiltonian, for instance describing finite-range interactions, but it is not clear whether it holds more generally. The role of backscattering in a TLL connected to leads [9,23,10] was found to be controlled by the Fabry-Perot dynamics recovered here by using equation (6), but the equivalence has to be checked and might be limited to linear transport. The boundary conditions (6) have the advantage to hold for any Hamiltonian in the low energy sector, and offer possibilities for future studies. One has to reformulate the bosonisation procedure to compute the correlation functions. Implementing equation (6) in a path integral formalism would give access to the nonlinear regime and current fluctuations. Conceptually, the scattering approach presented here can be extended to situations where linear response theory fails, and can be generalized to edge states in the fractional regime.

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Note added in proof

In a recent erratum [24], Egger and Grabert modified their boundary conditions for the TLL [19] by using self-consistent arguments [21]. Their corrected results agree with reference [4] and therefore with its present generalization.

References

1. R. Landauer, Philos. Mag. **21**, 863 (1970); M. Büttiker, Y. Imry, R. Landauer, S. Pinhas, Phys. Rev. B **31**, 6207 (1985).
2. M. Büttiker, A. Prêtre, H. Thomas, Phys. Rev. Lett. **70**, 4114 (1993).
3. N. S. Wingreen, A.P. Jauho, Y. Meir, Phys. Rev. B **48**, R8487 (1993-I); C. Bruder, H. Scholler, Phys. Rev. Lett. **72**, 1076 (1994).
4. I. Safi, H.J. Schulz, Phys. Rev. B **52**, R17040 (1995).
5. D. Maslov, M. Stone, Phys. Rev. B **52**, R5539 (1995); V.V. Ponomarenko, *ibid*, R8666.
6. A. Y. Alekseev, V. Cheianov, J. Fröhlich, Phys. Rev. B **54**, R17320 (1996); A. Shimizu, J. Phys. Soc. Jap. **65**, 1162 (1996); The first argument by Y. Oreg, A.M. Finkel'stein, Phys. Rev. B **54**, R14265 (1996) is similar [22], but is different from the second argument proposed also by A. Kawabata, J. Phys. Soc. Jap. **65**, 30 (1996), and based on Kubo formula without reservoirs; this yields in general a different conductance [7].
7. I. Safi, Phys. Rev. B **55**, R7331 (1997).
8. A. Alekseev, V. Cheianov, J. Fröhlich, cond-mat/9803346.
9. I. Safi, H.J. Schulz, in *Quantum Transport in Semiconductor Submicron Structures*, edited by B. Kramer (Kluwer Academic Press, Dordrecht, 1995); in *Correlated Fermions and Transport in Mesoscopic Systems*, edited by T. Martin, G. Montambaux, J.T. Van (Editions Frontières, Gif-sur-Yvette, 1996); Phys. Rev. B, **59**, 3040 (1999).
10. I. Safi-Taktak, Ph.D. thesis, Laboratoire de Physique des Solides, 1996; Ann. Phys. Fr. **22**, 463 (1997).
11. Ya.M. Blanter, F.W.J. Hekking, M. Büttiker, Phys. Rev. Lett. **81**, 1749 (1998).
12. V.V. Ponomarenko, Phys. Rev. B **52**, R8666 (1995); V.A. Sablikov and B.S. Shchamkhalova, JETP Lett. **66**, 41 (1997).
13. C.L. Kane, M.P.A. Fisher, Phys. Rev. B **52**, 17393 (1995).
14. C. de C. Chamon, E. Fradkin, Phys. Rev. B **55**, 4534 (1997-II).
15. N.P. Sandler, C. de C. Chamon, E. Fradkin, Phys. Rev. B **57**, 12324 (1998).
16. S. Tomonaga, Prog. Theor. Phys. **5**, 544 (1950); D.C. Mattis, E.H. Lieb, J. Math. Phys. **6**, 304 (1965).
17. For the specific case of a quadratic Hamiltonian, and in equilibrium, see also reference [22].
18. For a quadratic Hamiltonian, this result was shown in references [4,7], and justifies the hypothesis adopted in references [6].
19. R. Egger, H. Grabert, Phys. Rev. Lett. **77**, 538 (1996).
20. X.G. Wen, Phys. Rev. Lett. **64**, 2206 (1990).
21. R. Egger, H. Grabert, Phys. Rev. Lett. **79**, 3463 (1997).
22. Y. Oreg, A.M. Finkel'stein, Phys. Rev. Lett. **74**, 3668 (1993).
23. A. Furusaki, N. Nagaosa, Phys. Rev. B **54**, R5239 (1996). See also D. Maslov, Phys. Rev. B **52**, R14368 (1995), but where results are restricted to the bulk [10].
24. R. Egger, H. Grabert, Phys. Rev. Lett. **80**, 2255 (1998).